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Equi-Inclination Weissenberg Intensity Correction Factors for Absorption in Spheres and Cylinders, and for Crystal Monochromatized Radiation

BY W. L. BOND

Bell Telephone Laboratories, Inc., Murray Hill, New Jersey, U.S.A.

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Increased precision of intensity measurements through the use of counting methods calls for more careful corrections. A new absorption correction table is presented for cylinders with μR running from 0 to 8 by tenths and on to 20 by units; also a table for spheres, μR running from 0 to 10 by tenths. In both tables θ runs from 0° to 90° in 5° steps. For cylinders, the correction factor for upper levels is gotten by entering the table with $\mu R \sec \nu$ in place of μR and $\Upsilon/2$ in place of θ, then multiply this value by $\cos \nu$. Tables are also given of the power series coefficients to be used in expanding absorption factors for very large cylinders. A combined polarization Lorentz correction factor is presented for the case of crystal monochromatized radiation.

Introduction

In recent years, there has been increasing emphasis on obtaining highly accurate structure amplitudes from single crystal X-ray data. In the course of work in these Laboratories on a single crystal automatic diffractometer (Bond, 1955; Benedict, 1955), several problems arose with regard to the conversion of intensities to structure amplitudes. These involved: (1) A Lorentz-polarization correction for monochromatized radiation; (2) expanding and improving the absorption tables for cylindrical and spherical crystals; and (3) deriving the method for absorption correction of upper level equi-inclination Weissenberg intensities. In particular, the latter has, as far as can be ascertained, been ignored. It is rather difficult to see how even for very small crystals containing light atoms, one can claim great accuracy (some claims have been 3%) for F's obtained say from layers with $\nu \cong 30°$ when such proper correction has not been made. As an example for $\mu R = 1$, $\mu = 20°$, $\Upsilon = 10°$ the absorption correction is 5·09 not 4·80 as is gotten from $\mu R = 1$, true $\theta = 20·6°$.

Absorption correction tables

Consider scattering from a very small element of volume ΔV bathed in a uniform monochromatic collimated beam of X-rays. Let the beam be of intensity I watts cm.$^{-2}$ and let the scattering power in a direction parallel to a vector $\mathbf{r}$ be ϱ_r watts per cm.^{3}

per incident watt per cm.^{2}. In these terms the total power scattered parallel to **r** is

$$P_r = I\varrho_r \Delta V \qquad (1)$$

watts. This is the power entering the radiation detector *RD*, Fig. 1. Now consider scattering from a large body

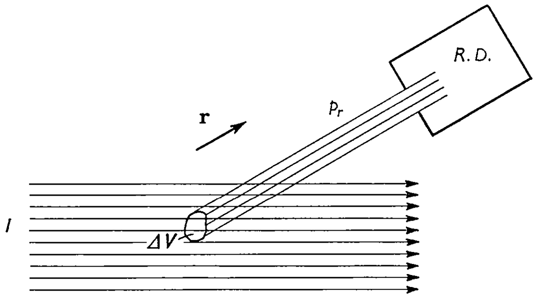

Fig. 1. Power scattered in a direction **r** from an element of volume.

of volume V which has a linear absorption coefficient μ, Fig. 2. Choose an element of volume, ΔV in V. The

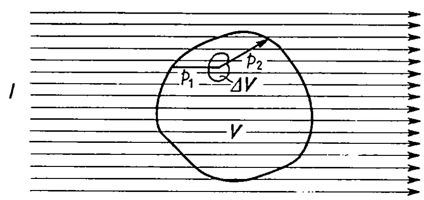

Fig. 2. Power scattered from a finite volume.

intensity I' falling on ΔV is $I' = I \exp(-\mu p_1)$ where p_1 is the path length, inside V of the incident radiation. The power leaving ΔV in the direction **r** is therefore $\Delta P_r' = I'\varrho_r \Delta V$ but this is further reduced by absorption along the path p_2 to $\Delta P_r = \Delta P_r' \exp(-\mu P_2)$. Whence the total power received by a properly placed radiation detector is

$$P_r = I\varrho_r \sum_V \exp(-\mu p)\Delta V \quad \text{where} \quad p = p_1 + p_2\,. \qquad (2)$$

Cylindrical specimens

Fig. 3 shows a circular cylinder receiving rays in a direction normal to its axis and scattering in a direction **r** also normal to this axis—the zero level case. Now let $\Delta V = w\Delta S$ where S is the cross sectional area of the cylinder, the length w being the width of the incident beam. Thus $P_r = I\varrho_r w \sum_S \exp(-\mu p)\Delta S$. Following Claassen (1930) we now replace S by s where:

$$S = \pi R^2 s$$

whence

$$P_r = \pi R^2 I \varrho_r w \sum_S \exp(-\mu p)\Delta s\,, \qquad (3)$$

where Δs is the fraction of the cross section in the area element which has path length p. For Bragg

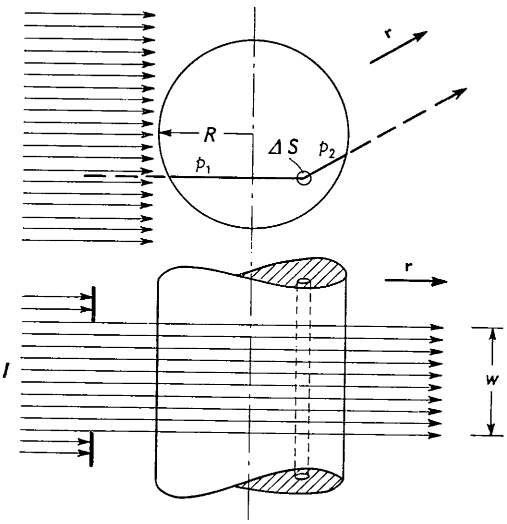

Fig. 3. Power scattered out of a cylinder.

angles θ of 0°, 22·5, 45°, 67·5 and 90° Claassen constructed curves of equal path length in a set of circles, the path lengths being $1/5R$, $2/5R$, etc. A planimeter was then used to measure the areas between successive curves and hence compute a set of Δs's. From this small set of Δs's with their associated path lengths he computed values for $A(\mu, \theta) = \Sigma \exp(-\mu g R)\Delta s$ where gR is the effective path length associated with Δs. Now making the proper substitution in (3)

$$P_r = \pi R^2 I \varrho_r w A_\theta \qquad (4)$$

or solving for ϱ_r

$$\varrho_r = P_r(\pi R^2 I w)^{-1} A_\theta^{-1}\,.$$

For relative values of ϱ_r the terms π, R^2, I and w may be omitted. These relative ϱ_r's are the intensities corrected only for absorption.

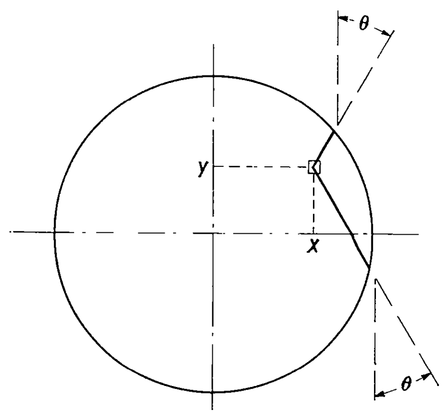

Fig. 4. Paths in a cylinder.

The accuracy of Claassen's method is limited by the necessity of drawing curves of equal path length and the measurement of areas by means of a plani-

Table 1. *Areas* $\triangle s$ *attributable to reduced paths g.*

0°		5°		10°		15°		20°		25°		30°		35°		40°		45°	
Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g
.00005	.05	.00033	0.050	.00094	0.050	.0019	.05	.00308	0.050	.00457	0.050	.0063	.05	.00828	0.050	.01046	0.050	.0129	.05
.00037	.15	.00076	0.150	.00164	0.150	.0028	.15	.00423	0.150	.00590	0.150	.0078	.15	.00986	0.150	.01210	0.150	.0144	.15
.00102	.25	.00135	0.250	.00248	0.250	.0039	.25	.00549	0.250	.00732	0.250	.0093	.25	.01146	0.250	.01371	0.250	.0160	.25
.00200	.35	.00224	0.350	.00349	0.350	.0051	.35	.00689	0.350	.00886	0.350	.0110	.35	.01314	0.350	.01539	0.350	.0177	.35
.00332	.45	.00360	0.450	.00465	0.450	.0065	.45	.00843	0.450	.01051	0.450	.0127	.45	.01489	0.450	.01712	0.450	.0194	.45
.00502	.55	.00527	0.550	.00600	0.550	.0080	.55	.01010	0.550	.01229	0.550	.0145	.55	.01672	0.550	.01890	0.550	.0210	.55
.00713	.65	.00734	0.650	.00760	0.650	.0097	.65	.01193	0.650	.01419	0.650	.0164	.65	.01861	0.650	.02072	0.650	.0227	.65
.00968	.75	.00985	0.750	.01038	0.750	.0116	.75	.01393	0.750	.01623	0.750	.0185	.75	.02059	0.750	.02260	0.750	.0244	.75
.01273	.85	.01285	0.850	.01323	0.850	.0137	.85	.01612	0.850	.01844	0.850	.0206	.85	.02266	0.850	.02453	0.850	.0262	.85
.01635	.95	.01642	0.950	.01663	0.950	.0160	.95	.01852	0.950	.02081	0.950	.0229	.95	.02483	0.950	.02652	0.950	.0280	.95
.02065	1.05	.02064	1.050	.02064	1.050	.0202	1.05	.02116	1.050	.02340	1.050	.0254	1.05	.02712	1.050	.02859	1.050	.0298	1.05
.02577	1.15	.02568	1.150	.02542	1.150	.0250	1.15	.02411	1.150	.02623	1.150	.0280	1.15	.02953	1.150	.03074	1.150	.0317	1.15
.03191	1.25	.03171	1.250	.03113	1.250	.0302	1.25	.02741	1.250	.02935	1.250	.0309	1.25	.03211	1.250	.03298	1.250	.0335	1.25
.03939	1.35	.03905	1.350	.03806	1.350	.0366	1.35	.03333	1.350	.03284	1.350	.0341	1.35	.03488	1.350	.03534	1.350	.0355	1.35
.04869	1.45	.04816	1.450	.04664	1.450	.0443	1.45	.04141	1.450	.03681	1.450	.0376	1.45	.03790	1.450	.03785	1.450	.0375	1.45
.06068	1.55	.05989	1.550	.05761	1.550	.0541	1.55	.04979	1.550	.04144	1.550	.0416	1.55	.04123	1.550	.04054	1.550	.0396	1.55
.07698	1.65	.07578	1.650	.07234	1.650	.0671	1.65	.06072	1.650	.05099	1.650	.0462	1.65	.04500	1.650	.04348	1.650	.0417	1.65
.10138	1.75	.09942	1.750	.09388	1.750	.0857	1.75	.07590	1.750	.06561	1.750	.0519	1.75	.04944	1.750	.04678	1.750	.0440	1.75
.14587	1.85	.14195	1.850	.13113	1.850	.1154	1.85	.09962	1.850	.08320	1.850	.0595	1.85	.05498	1.850	.05065	1.850	.0465	1.85
.39101	1.95	.31101	1.950	.24577	1.950	.1967	1.95	.15194	1.950	.11805	1.950	.0880	1.95	.06495	1.950	.05592	1.950	.0492	1.95
		.08671	2.004	.17033	2.015	.2454	2.036	.24373	2.050	.17339	2.050	.1243	2.05	.08164	2.050	.06257	2.050	.0511	2.05
								.07215	2.114	.18668	2.150	.1318	2.15	.08641	2.150	.06372	2.150	.0514	2.15
										.01292	2.203	.1468	2.25	.09563	2.250	.06503	2.250	.0514	2.25
												.0140	2.305	.11033	2.350	.06641	2.350	.0512	2.35
														.04779	2.421	.06849	2.450	.0506	2.45
																.07912	2.550	.0495	2.55
																.00974	2.605	.0477	2.65
																		.0442	2.75
																		.0111	2.814

50°		55°		60°		65°		70°		75°		80°		85°		90°	
Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g	Δs	g
.01530	.0500	.01786	.0500	.0204	.05	.02303	.0500	.02551	.0500	.0278	.05	.02978	.0500	.03126	.0500	.03183	.05
.01686	.150	.01935	.150	.0217	.15	.02426	.150	.02657	.150	.0285	.15	.03040	.150	.03168	.150	.03181	.15
.01838	.250	.02074	.250	.0230	.25	.02526	.250	.02731	.250	.0291	.25	.03057	.250	.03151	.250	.03177	.25
.01994	.350	.02217	.350	.0243	.35	.02632	.350	.02814	.350	.0297	.35	.03089	.350	.03155	.350	.03171	.35
.02150	.450	.02357	.450	.0256	.45	.02733	.450	.02891	.450	.0302	.45	.03112	.450	.03152	.450	.03163	.45
.02305	.550	.02496	.550	.0267	.55	.02828	.550	.02961	.550	.0307	.55	.03128	.550	.03146	.550	.03153	.55
.02459	.650	.02631	.650	.0279	.65	.02917	.650	.03024	.650	.0310	.65	.03124	.650	.03137	.650	.03141	.65
.02619	.750	.02764	.750	.0289	.75	.03000	.750	.03079	.750	.0313	.75	.03133	.750	.03127	.750	.03126	.75
.02768	.850	.02894	.850	.0300	.85	.03076	.850	.03127	.850	.0314	.85	.03126	.850	.03113	.850	.03110	.85
.02922	.950	.03021	.950	.0310	.95	.03145	.950	.03165	.950	.0315	.95	.03118	.950	.03097	.950	.03092	.95
.03075	1.05	.03145	1.05	.0319	1.05	.03207	1.05	.03195	1.05	.0315	1.05	.03105	1.05	.03078	1.05	.03071	1.05
.03228	1.15	.03265	1.15	.0328	1.15	.03259	1.15	.03214	1.15	.0314	1.15	.03089	1.15	.03057	1.15	.03049	1.15
.03381	1.25	.03381	1.25	.0335	1.25	.03302	1.25	.03221	1.25	.0313	1.25	.03070	1.25	.03034	1.25	.03024	1.25
.03533	1.35	.03491	1.35	.0342	1.35	.03333	1.35	.03217	1.35	.0312	1.35	.03048	1.35	.03008	1.35	.02996	1.35
.03684	1.45	.03595	1.45	.0348	1.45	.03350	1.45	.03205	1.45	.0310	1.45	.03022	1.45	.02979	1.45	.02967	1.45
.03834	1.55	.03692	1.55	.0353	1.55	.03351	1.55	.03189	1.55	.0307	1.55	.02994	1.55	.02948	1.55	.02934	1.55
.03982	1.65	.03777	1.65	.0356	1.65	.03338	1.65	.03168	1.65	.0305	1.65	.02962	1.65	.02914	1.65	.02899	1.65
.04126	1.75	.03845	1.75	.0356	1.75	.03320	1.75	.03142	1.75	.0301	1.75	.02927	1.75	.02877	1.75	.02862	1.75
.04261	1.85	.03884	1.85	.0355	1.85	.03295	1.85	.03110	1.85	.0298	1.85	.02889	1.85	.02837	1.85	.02822	1.85
.04360	1.95	.03880	1.95	.0352	1.95	.03263	1.95	.03073	1.95	.0294	1.95	.02847	1.95	.02795	1.95	.02779	1.95
.04374	2.05	.03856	2.05	.0349	2.05	.03223	2.05	.03030	2.05	.0289	2.05	.02801	2.05	.02749	2.05	.02733	2.05
.04353	2.15	.03822	2.15	.0345	2.15	.03177	2.15	.02982	2.15	.0285	2.15	.02752	2.15	.02700	2.15	.02684	2.15
.04318	2.25	.03773	2.25	.0340	2.25	.03122	2.25	.02927	2.25	.0279	2.25	.02699	2.25	.02647	2.25	.02632	2.25
.04260	2.35	.03708	2.35	.0333	2.35	.03059	2.35	.02867	2.35	.0273	2.35	.02642	2.35	.02591	2.35	.02576	2.35
.04175	2.45	.03625	2.45	.0326	2.45	.02987	2.45	.02799	2.45	.0267	2.45	.02581	2.45	.02531	2.45	.02516	2.45
.04057	2.55	.03520	2.55	.0316	2.55	.02905	2.55	.02725	2.55	.0260	2.55	.02514	2.55	.02466	2.55	.02452	2.55
.03893	2.65	.03390	2.65	.0306	2.65	.02813	2.65	.02642	2.65	.0252	2.65	.02443	2.65	.02398	2.65	.02384	2.65
.03665	2.75	.03229	2.75	.0293	2.75	.02708	2.75	.02551	2.75	.0244	2.75	.02366	2.75	.02324	2.75	.02311	2.75
.03331	2.85	.03027	2.85	.0278	2.85	.02590	2.85	.02451	2.85	.0235	2.85	.02283	2.85	.02245	2.85	.02233	2.85
.02765	2.95	.02767	2.95	.0260	2.95	.02456	2.95	.02339	2.95	.0226	2.95	.02194	2.95	.02159	2.95	.02150	2.95
.01064	3.032	.02420	3.05	.0240	3.05	.02303	3.05	.02215	3.05	.0215	3.05	.02097	3.05	.02067	3.05	.02059	3.05
		.01907	3.15	.0214	3.15	.02126	3.15	.02077	3.15	.0203	3.15	.01991	3.15	.01968	3.15	.01961	3.15
		.08087	3.238	.0181	3.25	.01919	3.25	.01920	3.25	.0190	3.25	.01875	3.25	.01859	3.25	.01855	3.25
				.0135	3.35	.01669	3.35	.01739	3.35	.0175	3.35	.01747	3.35	.01740	3.35	.01739	3.35
				.0045	3.432	.01350	3.45	.01528	3.45	.0158	3.45	.01603	3.45	.01608	3.45	.01610	3.45
						.008834	3.55	.01268	3.55	.0139	3.55	.01440	3.55	.01460	3.55	.01466	3.55
						.0008847	3.613	.009190	3.65	.0116	3.65	.01250	3.65	.01289	3.65	.01301	3.65
								.002690	3.729	.0085	3.75	.01016	3.75	.01085	3.75	.01106	3.75
										.0027	3.832	.006952	3.85	.003247	3.85	.00859	3.85
												.001226	3.92	.003737	3.942	.00473	3.95

meter. To avoid this graphic approach the IBM ♯650 has been used to make the computations. Consider a circle, (Fig. 4) about the origin and a beam entering from the upper right reflecting from an element of an area about the point x, y, hence leaving towards the lower right, the deflection angle being 2θ. The path length inside the circle is:

$$p = R.\{1-(x\cos\theta - y\sin\theta)^2\}^{\frac{1}{2}} + R\{1-(x\cos\theta + y\sin\theta)^2\}^{\frac{1}{2}} - 2xR\sin\theta\dots, \quad (5)$$

where x and y are now fractions of R.

For θ values of 0, 5°, 10°, ..., 90° and path lengths $.1R$, $.2R$... an I.B.M. card Programmed Electronic Calculator solved equation (5) and integrated to give the areas of the regions. If a mean path length is attributed to each region the results are as in Table 1. As an example of the use of this table consider the case $\mu R = 2$, $\theta = 0$. Now

$$A = 0{\cdot}00005\exp(-2\times 0{\cdot}05) + 0{\cdot}0037\exp(-2\times 0{\cdot}15) + 0{\cdot}00102\exp(-2\times 0{\cdot}25) + \dots + 0{\cdot}39101\exp(-2\times 1{\cdot}95) + \dots .$$

It is more convenient in use to replace A by the correction factor A^*, i.e. the number by which the observed intensity should be multiplied to get the 'absorption free' intensity. In this way Table 2 has been prepared.

For values of $\mu R > 8$, the above method gives erroneous results since too much of the scattering is from the surface of the element of area of the shortest path length. For μR large then, the first zones are too large and should be subdivided to give a better evaluation of A. This subdivision can be approximated by finding s (see (3)) as a function of g near $s = 0$. If $s = \alpha g + \beta g^2 + \gamma g^3\dots$ then

$$ds = (\alpha + 2\beta g + 3\gamma g^2\dots)\,dg$$

Table 2. *Absorption correction factors A^* for X-ray intensities*

Cylinders of radius R

μR	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	μR
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
.1	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	.1
.2	1.40	1.40	1.40	1.40	1.40	1.40	1.39	1.39	1.39	1.39	1.39	1.38	1.38	1.38	1.38	1.38	1.37	1.37	1.37	.2
.3	1.65	1.65	1.65	1.65	1.65	1.64	1.64	1.63	1.63	1.62	1.62	1.61	1.61	1.60	1.59	1.59	1.59	1.59	1.59	.3
.4	1.95	1.95	1.95	1.94	1.94	1.93	1.92	1.91	1.90	1.89	1.87	1.86	1.85	1.84	1.83	1.82	1.82	1.81	1.81	.4
.5	2.29	2.29	2.29	2.28	2.27	2.26	2.24	2.22	2.20	2.18	2.16	2.13	2.12	2.10	2.08	2.07	2.06	2.05	2.05	.5
.6	2.69	2.69	2.69	2.67	2.65	2.63	2.60	2.57	2.53	2.50	2.47	2.43	2.40	2.37	2.35	2.33	2.31	2.30	2.30	.6
.7	3.16	3.16	3.15	3.13	3.09	3.05	3.01	2.96	2.91	2.85	2.80	2.75	2.71	2.66	2.63	2.60	2.58	2.56	2.56	.7
.8	3.70	3.70	3.68	3.65	3.60	3.54	3.47	3.39	3.32	3.24	3.16	3.09	3.03	2.97	2.92	2.88	2.85	2.84	2.83	.8
.9	4.33	4.33	4.30	4.24	4.17	4.08	3.98	3.87	3.76	3.65	3.55	3.46	3.37	3.29	3.23	3.18	3.14	3.11	3.11	.9
1.0	5.06	5.05	5.01	4.92	4.81	4.68	4.54	4.39	4.24	4.10	3.97	3.84	3.73	3.63	3.55	3.48	3.43	3.40	3.39	1.0
1.1	5.90	5.88	5.81	5.69	5.54	5.36	5.16	4.96	4.76	4.58	4.40	4.24	4.10	3.97	3.87	3.79	3.73	3.69	3.68	1.1
1.2	6.86	6.84	6.74	6.57	6.35	6.10	5.84	5.57	5.32	5.08	4.86	4.66	4.49	4.33	4.20	4.11	4.03	3.98	3.97	1.2
1.3	7.96	7.93	7.79	7.55	7.25	6.92	6.58	6.23	5.91	5.61	5.34	5.09	4.88	4.70	4.54	4.43	4.34	4.28	4.27	1.3
1.4	9.23	9.18	8.97	8.65	8.25	7.82	7.37	6.94	6.53	6.16	5.83	5.54	5.29	5.07	4.89	4.75	4.65	4.58	4.57	1.4
1.5	10.7	10.6	10.3	9.88	9.35	8.79	8.22	7.68	7.19	6.74	6.35	6.00	5.71	5.45	5.24	5.08	4.96	4.89	4.87	1.5
1.6	12.3	12.2	11.8	11.2	10.6	9.84	9.13	8.47	7.87	7.34	6.87	6.47	6.13	5.84	5.60	5.42	5.28	5.19	5.17	1.6
1.7	14.2	14.0	13.5	12.7	11.9	11.0	10.1	9.30	8.58	7.96	7.42	6.95	6.57	6.23	5.96	5.75	5.60	5.50	5.48	1.7
1.8	16.3	16.0	15.4	14.4	13.3	12.2	11.1	10.2	9.32	8.59	7.97	7.45	7.01	6.63	6.33	6.09	5.92	5.81	5.78	1.8
1.9	18.6	18.3	17.5	16.2	14.8	13.5	12.2	11.1	10.1	9.25	8.54	7.94	7.45	7.03	6.69	6.44	6.24	6.12	6.09	1.9
2.0	21.3	20.9	19.8	18.2	16.5	14.8	13.3	12.0	10.9	9.91	9.12	8.45	7.91	7.44	7.06	6.78	6.56	6.43	6.40	2.0
2.1	24.2	23.7	22.3	20.3	18.2	16.2	14.5	12.9	11.7	10.6	9.71	8.97	8.36	7.84	7.44	7.13	6.89	6.75	6.71	2.1
2.2	27.5	26.9	25.1	22.6	20.1	17.7	15.7	13.9	12.5	11.3	10.3	9.49	8.82	8.26	7.81	7.47	7.22	7.06	7.02	2.2
2.3	31.2	30.4	28.1	25.1	22.0	19.3	16.9	14.9	13.3	12.0	10.9	10.0	9.29	8.67	8.19	7.82	7.54	7.38	7.33	2.3
2.4	35.3	34.2	31.4	27.7	24.1	20.9	18.2	16.0	14.2	12.7	11.5	10.5	9.76	9.09	8.57	8.17	7.87	7.69	7.64	2.4
2.5	39.8	38.5	34.9	30.5	26.2	22.5	19.5	17.0	15.0	13.4	12.1	11.1	10.2	9.51	8.95	8.53	8.20	8.01	7.96	2.5
2.6	44.7	43.1	38.7	33.4	28.5	24.2	20.8	18.1	15.9	14.2	12.8	11.6	10.7	9.93	9.33	8.88	8.53	8.33	8.27	2.6
2.7	50.1	48.1	42.8	36.5	30.8	26.0	22.2	19.2	16.8	14.9	13.4	12.2	11.2	10.4	9.72	9.23	8.87	8.64	8.58	2.7
2.8	56.1	53.5	47.2	39.7	33.2	27.8	23.6	20.3	17.7	15.6	14.0	12.7	11.7	10.8	10.1	9.59	9.20	8.96	8.90	2.8
2.9	62.5	59.4	51.8	43.1	35.7	29.6	25.0	21.4	18.6	16.4	14.6	13.2	12.1	11.2	10.5	9.95	9.53	9.28	9.21	2.9
3.0	69.5	65.8	56.7	46.6	38.2	31.5	26.4	22.5	19.5	17.1	15.3	13.8	12.6	11.6	10.9	10.3	9.86	9.60	9.53	3.0
3.1	77.2	72.6	61.8	50.3	40.8	33.4	27.9	23.7	20.4	17.9	15.9	14.3	13.1	12.1	11.3	10.7	10.2	9.92	9.84	3.1
3.2	85.4	79.9	67.3	54.0	43.5	35.4	29.3	24.8	21.4	18.7	16.6	14.9	13.6	12.5	11.7	11.0	10.5	10.2	10.2	3.2
3.3	94.2	87.6	72.9	57.9	46.2	37.3	30.8	26.0	22.3	19.4	17.2	15.5	14.1	12.9	12.0	11.4	10.9	10.6	10.5	3.3
3.4	104	95.9	78.9	61.9	48.9	39.3	32.3	27.1	23.2	20.2	17.9	16.0	14.6	13.4	12.4	11.7	11.2	10.9	10.8	3.4
3.5	114	105	85.0	65.9	51.7	41.3	33.8	28.3	24.2	21.0	18.5	16.6	15.1	13.8	12.8	12.1	11.5	11.2	11.1	3.5
3.6	125	114	91.4	70.1	54.6	43.3	35.3	29.5	25.1	21.7	19.2	17.1	15.6	14.2	13.2	12.5	11.9	11.5	11.4	3.6
3.7	136	124	98.0	74.4	57.4	45.4	36.9	30.7	26.1	22.5	19.8	17.7	16.1	14.7	13.6	12.8	12.2	11.8	11.7	3.7
3.8	149	134	105	78.7	60.4	47.5	38.4	31.9	27.0	23.3	20.5	18.3	16.6	15.1	14.0	13.2	12.6	12.2	12.1	3.8
3.9	162	145	112	83.1	63.3	49.5	39.9	33.1	28.0	24.1	21.2	18.8	17.1	15.6	14.4	13.6	12.9	12.5	12.4	3.9
4.0	175	156	119	87.6	66.3	51.6	41.5	34.3	28.9	24.9	21.8	19.4	17.6	16.0	14.8	13.9	13.2	12.8	12.7	4.0
4.1	190	168	127	92.1	69.3	53.8	43.1	35.5	29.9	25.7	22.5	20.0	18.1	16.4	15.2	14.3	13.6	13.1	13.0	4.1
4.2	206	180	134	96.7	72.3	55.9	44.6	36.7	30.9	26.5	23.2	20.6	18.6	16.9	15.6	14.6	13.9	13.5	13.3	4.2
4.3	222	193	142	101	75.4	58.0	46.2	37.9	31.8	27.3	23.8	21.1	19.1	17.3	16.0	15.0	14.3	13.8	13.7	4.3
4.4	239	206	150	106	78.5	60.2	47.8	39.1	32.8	28.1	24.5	21.7	19.6	17.8	16.4	15.4	14.6	14.1	14.0	4.4
4.5	257	220	158	111	81.6	62.3	49.4	40.4	33.8	28.9	25.2	22.3	20.1	18.2	16.8	15.7	14.9	14.4	14.3	4.5
4.6	275	234	166	116	84.7	64.5	51.0	41.6	34.8	29.7	25.8	22.9	20.6	18.7	17.2	16.1	15.3	14.8	14.6	4.6
4.7	295	249	175	121	87.8	66.7	52.6	42.8	35.8	30.5	26.5	23.4	21.1	19.1	17.6	16.5	15.6	15.1	14.9	4.7
4.8	316	264	183	125	91.0	68.9	54.2	44.1	36.7	31.3	27.2	24.0	21.6	19.6	18.0	16.9	16.0	15.4	15.3	4.8
4.9	337	280	192	130	94.2	71.1	55.8	45.3	37.7	32.1	27.9	24.6	22.1	20.0	18.4	17.2	16.3	15.7	15.6	4.9
5.0	359	296	200	135	97.4	73.3	57.5	46.6	38.7	32.9	28.6	25.2	22.6	20.5	18.8	17.6	16.7	16.1	15.9	5.0
5.1	383	313	209	140	101	75.5	59.1	47.8	39.7	33.7	29.2	25.8	23.1	20.9	19.2	18.0	17.0	16.4	16.2	5.1
5.2	407	330	218	145	104	77.8	60.7	49.1	40.7	34.5	29.9	26.3	23.6	21.4	19.6	18.3	17.3	16.7	16.6	5.2
5.3	432	348	228	150	107	80.0	62.4	50.3	41.7	35.3	30.6	26.9	24.1	21.8	20.0	18.7	17.7	17.1	16.9	5.3
5.4	458	366	237	156	110	82.3	64.0	51.6	42.7	36.1	31.3	27.5	24.6	22.3	20.4	19.1	18.0	17.4	17.2	5.4
5.5	485	384	246	161	114	84.5	65.7	52.9	43.7	37.0	32.0	28.1	25.2	22.7	20.9	19.5	18.4	17.7	17.5	5.5
5.6	513	403	255	166	117	86.8	67.3	54.1	44.7	37.8	32.7	28.7	25.7	23.2	21.3	19.8	18.7	18.0	17.9	5.6
5.7	542	422	265	171	120	89.1	69.0	55.4	45.7	38.6	33.4	29.3	26.2	23.6	21.7	20.2	19.1	18.4	18.2	5.7
5.8	573	442	275	176	124	91.4	70.7	56.7	46.8	39.4	34.0	29.9	26.7	24.1	22.1	20.6	19.4	18.7	18.5	5.8
5.9	604	462	284	182	127	93.6	72.4	58.0	47.8	40.3	34.7	30.5	27.2	24.5	22.5	21.0	19.8	19.0	18.8	5.9
6.0	636	483	294	187	130	95.9	74.0	59.2	48.8	41.1	35.4	31.0	27.7	26.0	22.9	21.3	20.1	19.4	19.2	6.0
6.1	670	504	304	192	134	98.3	75.7	60.5	49.8	41.9	36.1	31.6	28.3	25.4	23.3	21.7	20.5	19.7	19.5	6.1
6.2	704	525	314	198	137	101	77.4	61.8	50.8	42.7	36.8	32.2	28.8	25.9	23.7	22.1	20.8	20.0	19.8	6.2
6.3	740	547	324	203	140	103	79.1	63.1	51.9	43.6	37.5	32.8	29.3	26.4	24.1	22.5	21.2	20.4	20.2	6.3
6.4	777	569	334	209	144	105	80.8	64.4	52.9	44.4	38.2	33.4	29.8	26.8	24.6	22.8	21.5	20.7	20.5	6.4
6.5	814	591	345	214	147	108	82.5	65.7	53.9	45.2	38.9	34.0	30.3	27.3	25.0	23.2	21.9	21.0	20.8	6.5
6.6	853	614	355	219	151	110	84.2	67.0	54.9	46.1	39.6	34.6	30.9	27.7	25.4	23.6	22.2	21.4	21.1	6.6
6.7	894	638	365	225	154	112	85.9	68.3	56.0	46.9	40.3	35.2	31.4	28.2	25.8	24.0	22.6	21.7	21.5	6.7
6.8	935	661	376	231	158	115	87.6	69.6	57.0	47.8	41.0	35.8	31.9	28.7	26.2	24.4	23.0	22.1	21.8	6.8
6.9	978	685	386	236	161	117	89.4	70.9	58.0	48.6	41.7	36.4	32.4	29.1	26.6	24.8	23.3	22.4	22.1	6.9
7.0	1022	710	397	242	165	119	91.1	72.2	59.1	49.4	42.4	37.0	33.0	29.6	27.0	25.1	23.7	22.7	22.5	7.0
7.1	1067	735	408	247	168	122	92.8	73.6	60.1	50.3	43.1	37.6	33.5	30.1	27.5	25.5	24.0	23.1	22.8	7.1
7.2	1113	760	418	253	172	124	94.6	74.9	61.2	51.1	43.8	38.2	34.0	30.5	27.9	25.9	24.4	23.4	23.1	7.2
7.3	1161	785	429	259	175	127	96.3	76.2	62.2	52.0	44.6	38.8	34.6	31.0	28.3	26.3	24.7	23.8	23.5	7.3
7.4	1210	811	440	264	179	129	98.1	77.5	63.3	52.8	45.3	39.5	35.1	31.5	28.7	26.7	25.1	24.1	23.8	7.4
7.5	1260	837	451	270	182	131	99.8	78.9	64.3	53.7	46.0	40.1	35.6	31.9	29.2	27.1	25.5	24.4	24.1	7.5
7.6	1311	864	462	276	186	134	102	80.2	65.4	54.5	46.7	40.7	36.1	32.4	29.6	27.4	25.8	24.8	24.5	7.6
7.7	1364	891	473	281	190	136	103	81.5	66.4	55.4	47.4	41.3	36.7	32.9	30.0	27.8	26.2	25.1	24.8	7.7
7.8	1418	918	484	287	193	139	105	82.9	67.5	56.2	48.1	41.9	37.2	33.3	30.4	28.2	26.5	25.5	25.2	7.8
7.9	1474	945	495	293	197	141	107	84.2	68.5	57.1	48.9	42.5	37.8	33.8	30.8	28.6	26.9	25.8	25.5	7.9
8	1530	974	507	299	200	144	109	85.5	69.6	58.0	49.5	43.1	38.3	34.3	31.3	29.0	27.3	26.1	25.8	8
9	2200	1270	623	357	236	167	126	98.1	79.6	65.6	56.3	48.7	43.2	38.6	35.2	32.5	30.6	29.2	28.9	9
10	3050	1600	743	417	272	191	143	111	89.7	74.2	63.0	54.3	48.2	43.0	39.1	36.1	33.9	32.4	31.9	10
11	4080	1950	868	478	300	215	160	124	99.7	82.4	69.8	60.0	53.2	47.4	43.1	39.7	37.3	35.5	35.0	11
12	5320	2320	995	541	345	240	178	137	110	90.6	76.5	65.6	58.2	51.7	47.0	43.3	40.6	38.7	38.1	12
13	6790	2710	1130	604	381	264	195	150	120	98.8	83.4	71.3	63.2	56.1	50.9	46.9	43.9	41.9	41.2	13
14	8500	3110	1260	667	419	289	212	163	130	107	90.1	77.0	68.2	60.5	54.9	50.5	47.2	45.1	44.3	14
15	10500	3530	1400	731	457	313	230	176	140	115	97.0	82.7	73.2	64.8	58.8	54.1	50.6	48.3	47.5	15
16	12700	3910	1540	795	495	338	249	189	151	124	104	88.4	78.2	69.2	62.8	57.7	54.0	51.5	50.6	16
17	15300	4410	1695	860	533	362	265	203	161	132	111	94.0	83.2	73.6	66.8	61.3	57.3	54.6	50.7	17
18	18300	4860	1820	925	572	387	282	216	171	140	118	99.7	88.2	78.0	70.7	64.9	60.7	57.8	56.8	18
19	21400	5320	1960	491	610	412	300	229	181	148	124	105	93.2	82.4	74.7	68.5	64.1	61.0	60.0	19
20	25000	5790	2100	1060	649	438	317	242	192	157	131	111	98.4	86.8	78.7	72.3	67.6	64.3	63.1	20
μR	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	μR

and

$$A = \int \exp(-\mu g R)\,ds$$

becomes

$$A = \alpha\int_0^\infty \exp(-\mu g R)\,dg + 2\beta\int_0^\infty \exp(-\mu g R)\,g\,dg + 3\gamma\int_0^\infty \exp(-\mu g R)\,g^2 dg$$

etc. or

$$A = \alpha/\mu R + 2\beta/(\mu R)^2 + 6\gamma/(\mu R)^3 + 24\delta/(\mu R)^4 + 120\varepsilon/(\mu R)^5 + 720\zeta/(\mu r)^6 \text{ etc.}$$

If we plot, from Table 1, the sum of the first i values of Δs against the ith value of $g(i = 1, 2, 3 \ldots)$ we have a graph of s as a function of g. Empirical means can be used to find from these data a power series expressing s as a function of g. The first coefficient α can be computed exactly and using this value the

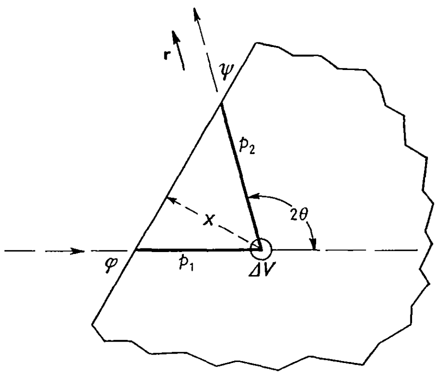

Fig. 5. Path lengths in a semi-infinite solid.

next few coefficients may be found empirically. Consider scattering from an element of volume ΔV inside a semi infinite solid, Fig. 5,

$$\Delta P_r = I\varrho_r \exp\left[-\mu(p_1+p_2)\right]\Delta V$$

but $x = p_1 \sin \Phi = p_2 \sin (2\theta-\Phi)$ so

$$\Delta P_r = I\varrho_r \exp\left[-\mu p_1\left\{1+\frac{\sin\Phi}{\sin(2\theta-\Phi)}\right\}\right]\Delta V\,.$$

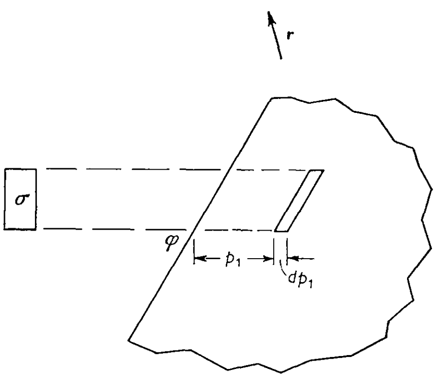

Fig. 6. Scattering from an element of volume in a semi-infinite solid.

A beam of cross sectional area σ (see Fig. 6) sends power

$$P_r = I\varrho_r\sigma\int^{\infty}\exp\left[-\mu p_1\left\{1+\frac{\sin\Phi}{\sin(2\theta-\Phi)}\right\}\right]dp_1$$

in the direction **r**.

Integrating:

$$P_r = \frac{I\varrho_r\sigma\sin(2\theta-\Phi)}{\mu\left[\sin\Phi+\sin(2\theta-\Phi)\right]}\,.$$

Now consider the effective scattering section, of the cylinder, Fig. 7, the part between $\Phi = 0$ and $\Phi = 2\theta$. We can compute the power sent parallel to **r** by an infinity of blocks of length $R\Delta\Phi$. Here σ of the preceding equation is replaced by $\Delta\sigma = wR\sin\Phi\Delta\Phi$ so that the power increment is

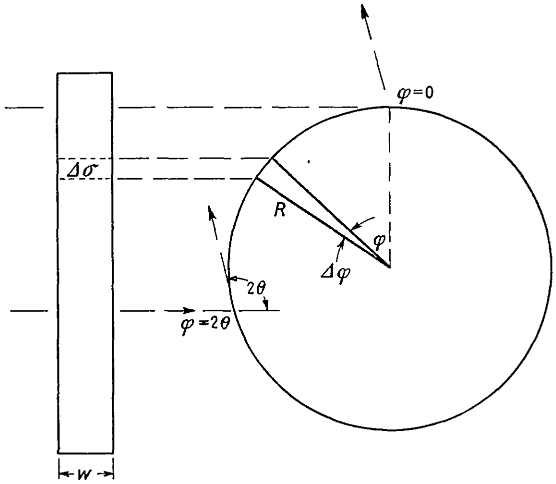

Fig. 7. Scattering from a cylinder of large μ.

$$\Delta P_v = \frac{I\varrho_r wR}{\mu}\cdot\frac{\sin\Phi\sin(2\theta-\Phi)}{\sin\Phi+\sin(2\theta-\Phi)}\Delta\Phi$$

and

$$P_v = \frac{I\varrho_r wR}{\mu}\int_{\Phi+0}^{2\theta}\frac{\sin\Phi\sin(2\theta-\Phi)}{\sin\Phi+\sin(2\theta-\Phi)}\,d\Phi\,.$$

Integrating

$$P_r = \frac{I\varrho_r wR}{\mu}\left\{1-\frac{\cos^2\theta}{\sin\theta}\ln\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}.$$

Comparing this with equation (4)

$$A_\theta = \frac{1}{\pi\mu R}\left\{1-\frac{\cos^2\theta}{\sin\theta}\ln\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}\text{ for }\mu R\text{ large.}$$

Hence the coefficient of $(\mu R)^{-1}$ is

$$\pi^{-1}\left\{1-\frac{\cos^2\theta}{\sin\theta}\ln\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}.$$

Table 3. *Coefficients for 5° intervals of θ*

θ	α	2β	6γ	248	120ε		5040η
0	0	0	0·3183	0	0·955	0	7·1
5	0·001615	0·0314	0·0648	0·835	0·14		
10	0·00642	0·053	0·179	0·057			
15	0·01442	0·086	0·061	0·33			
20	0·0254	0·105	0·080	0·13			
25	0·0394	0·123	0·073	−0·16			
30	0·0560	0·139	0				
35	0·0752	0·150	−0·022				
40	0·0966	0·159	−0·10				
45	0·1199	0·164	−0·16				
50	0·1448	0·16	−0·20				
55	0·1741	0·13	−0·30				
60	0·1984	0·11	−0·21				
65	0·226	0·10	−0·33				
70	0·250	0·105	−0·48				
75	0·274	0·08	−0·52				
80	0·295	0·064	−0·64				
85	0·3107	0·03	−0·5				
90	0·3183	0	−0·53				

For small θ, $\alpha = 2\theta^2/(3\pi)$. $A \doteqdot \alpha/(\mu R)+2\beta/(\mu^2R^2)\ldots.$

Table 3 gives the values of the coefficients for 5° intervals of θ. We can find A and hence A^* for intermediate values of θ by plotting A against $\sin^2\theta$ (Bradley, 1935).

Optimum size of cylinders for the zero level

Equation (4) indicates that the power received by the radiation detector is proportional to $(\mu R)^2 A\theta$. Plots of $(\mu R)^2/A^*$ show that for $\theta = 0°$ the optimum size is $\mu R = 1{\cdot}35$, for $\theta = 10°$ it is $\mu R = 1{\cdot}45$ and for $\theta = 20°$ it is $\mu R = 1{\cdot}9$. Above $\theta = 25°$ there is no optimum since $(\mu R)^2/A^*$ rises continuously from $\mu R = 0$ to $\mu R = \infty$ if $\theta > 25°$. Hence a practical optimum for cylinders is $\mu R = 1{\cdot}5$.

Absorption corrections for upper levels of the equi-inclination Weissenberg-cylindrical specimens

In the equi-inclination case, Fig. 8 both the on coming and the departing rays make an angle $90°-\nu$ with the cylinder axis. Hence the path lengths p_1 and p_2 of Fig. 3 become $p_1 \sec\nu$ and $p_2 \sec\nu$ in Fig. 8.

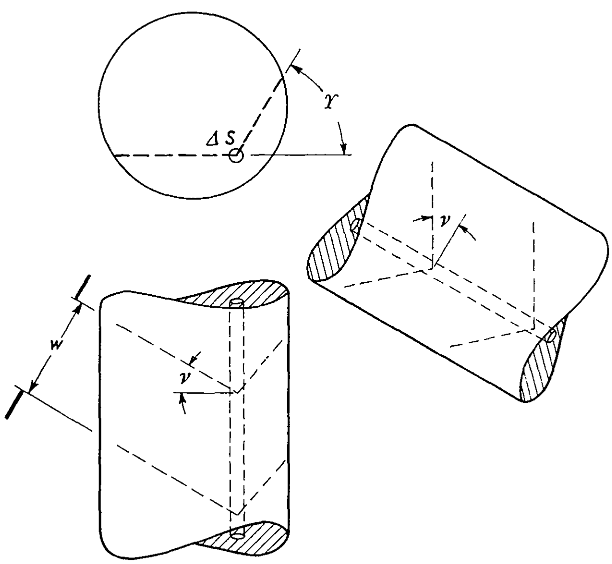

Fig. 8. Path lengths and angles for upper levels.

Also the volume element is $w \sec\nu\Delta S$. Hence the power received by a properly placed radiation detector is from equation (2):

$$P_r = I\varrho_r \sum_v \exp(-\mu \sec\nu . p)\Delta V$$

or

$$P_r = \pi R^2 I\varrho_r w \sec\nu \sum_s \exp[(-\mu\sec\nu)\varrho]\Delta s\,. \quad (6)$$

Setting $\mu' = \mu\sec\nu$ it is seen that the function under the summation is $A(\mu' R,\ \Upsilon/2)$. If ν goes to zero $\Upsilon/2$ becomes θ and μ' becomes μ so that $A(\mu' R,\ \Upsilon/2)$ goes to $A(\mu R, \theta)$. To compare reflections in the same level it is necessary only to compare the respective values of $P_v A^*(\mu' R,\ \Upsilon/2) = \varrho_v \times \text{const.}$ However if we wish to compare reflections in different levels we must use

$$\varrho_r \times \text{const.} = P_r \cos\nu\, A^*(\mu R \sec\nu,\ \Upsilon/2)\,.$$

Spherical specimens

If we take $V = 4/3\pi R^3 v$ we may re-write equation (2) as

$$P_r = 4/3\pi R^3 I \varrho_r \sum_r \exp(-\mu p)\Delta v\,.$$

A machine integration of $\sum_r \exp(-\mu p)\Delta v$ gives the data of Table 4. It is worth noting that for $\theta = 0$ and $\theta = 90°$ the equation is integrable. For $\theta = 0$ the absorption factor is

$$A = \frac{3}{2(\mu R)^3}\{\tfrac{1}{2} - \exp(-2\mu R)[\tfrac{1}{2} + \mu R + (\mu R)^2]\}\,.$$

While for $\theta = 90°$:

$$A = \frac{3}{4\mu R}\left\{\frac{1}{2} - \frac{1}{16(\mu R)^2}[1-(1+4\mu R)\exp(-4\mu R)]\,.\right.$$

Optimum size for spheres

Plotting $(\mu R)^3/A^*$ shows that there is no optimum size for spheres, the total reflected energy rises continously from $\mu R = 0$ to $\mu R = \infty$ for all values of θ.

The concept of 'optimum size' is misleading. The desirable condition is that the correction factor for $\theta = 0$ be not too different from the correction factor for $\theta = 90°$. This is met if $\mu R < 2$, when the ratio of front to back factors is less than 2·6.

The polarization correction

The polarization correction using a monochromator (Whittaker, 1952) can be combined with the Lorentz factor as follows: Whittakers result for the equi-inclination case may be written as a correction factor as

$$P = (1+\cos^2 2e)^{-1}\{\cos^2 2e(1-\cos^2\nu\sin^2\Upsilon) + 1 - \sin^2\nu\cos^2\nu(1+\cos\Upsilon)^2\}\,,$$

where e is the Bragg angle for the monochromator.

The Lorentz factor as adapted to the equi-inclination Weissenberg by Tunnel (1939) is $L = \cos^2\nu \sin\Upsilon$. Since $\cos\theta = \cos\nu\cos\Upsilon/2$ we can write the combined correction factor as

$$P_L = T\sin 2\theta \Big/ \left\{1 + \left(\frac{q-\sin^2\nu}{(1+q\cos^2\nu)}\right) \times (1+\cos 2\theta)^2 - \frac{2q}{1+q}(1+\cos 2\theta)\right\}\,,$$

where $q = \cos^2 2e$

and
$$T = (\sin^2\theta - \sin^2\nu)^{\frac{1}{2}}/\sin\theta\,.$$

Table 4. *Absorption correction factors A^* for X-ray intensities*

Spheres of radius R

μR	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	μR
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
.1	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	.1
.2	1.35	1.35	1.35	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	.2
.3	1.56	1.56	1.56	1.55	1.55	1.55	1.55	1.54	1.54	1.54	1.53	1.53	1.52	1.52	1.51	1.51	1.51	1.51	1.51	.3
.4	1.80	1.80	1.80	1.79	1.79	1.78	1.78	1.77	1.76	1.75	1.74	1.73	1.73	1.72	1.71	1.70	1.70	1.70	1.70	.4
.5	2.08	2.07	2.07	2.06	2.06	2.05	2.03	2.02	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	1.90	1.90	.5
.6	2.39	2.39	2.38	2.37	2.36	2.34	2.32	2.30	2.27	2.25	2.23	2.30	2.18	2.16	2.14	2.13	2.11	2.11	2.11	.6
.7	2.75	2.74	2.73	2.72	2.70	2.67	2.64	2.60	2.57	2.53	2.50	2.46	2.43	2.40	2.37	2.35	2.33	2.32	2.32	.7
.8	3.15	3.15	3.13	3.11	3.07	3.03	2.99	2.94	2.89	2.83	2.78	2.73	2.69	2.65	2.61	2.58	2.56	2.55	2.55	.8
.9	3.61	3.60	3.58	3.54	3.50	3.44	3.37	3.30	3.23	3.16	3.09	3.02	2.96	2.91	2.86	2.82	2.80	2.78	2.77	.9
1.0	4.12	4.11	4.08	4.03	3.96	3.88	3.79	3.70	3.63	3.50	3.41	3.33	3.25	3.18	3.12	3.07	3.02	3.02	3.01	1.0
1.1	4.70	4.69	4.64	4.57	4.48	4.37	4.25	4.12	3.99	3.87	3.75	3.64	3.55	3.46	3.39	3.33	3.28	3.26	3.25	1.1
1.2	5.35	5.33	5.27	5.17	5.05	4.90	4.74	4.57	4.41	4.25	4.11	.97	3.85	3.75	3.66	3.59	3.54	3.50	3.49	1.2
1.3	6.08	6.05	5.97	5.84	5.67	5.47	5.27	5.06	4.85	4.66	4.48	4.32	4.17	4.04	3.94	3.85	3.79	3.75	3.73	1.3
1.4	6.90	6.86	6.75	6.57	6.35	6.10	5.83	5.57	5.32	5.08	4.86	4.67	4.49	4.34	4.22	4.12	4.05	4.00	3.98	1.4
1.5	7.80	7.75	7.60	7.38	7.09	6.77	6.44	6.11	5.81	5.52	5.26	5.03	4.83	4.65	4.51	4.39	4.31	4.26	4.23	1.5
1.6	8.81	8.74	8.55	8.25	7.89	7.49	7.08	6.69	6.32	5.98	5.67	5.40	5.17	4.97	4.80	4.67	4.57	4.51	4.48	1.6
1.7	9.92	9.83	9.59	9.21	8.76	8.26	7.76	7.29	6.85	6.45	6.10	5.78	5.51	5.28	5.09	4.94	4.83	4.77	4.74	1.7
1.8	11.2	11.0	10.7	10.3	9.69	9.08	8.48	7.92	7.40	6.94	6.53	6.17	5.87	5.60	5.39	5.22	5.10	5.02	4.99	1.8
1.9	12.5	12.4	12.0	11.4	10.7	9.95	9.24	8.58	7.98	7.44	6.97	6.57	6.22	5.92	5.69	5.50	5.37	5.28	5.25	1.9
2.0	14.0	13.8	13.3	12.6	11.8	10.9	10.0	9.26	8.57	7.96	7.43	6.97	6.59	6.26	5.99	5.78	5.63	5.54	5.50	2.0
2.1	15.6	15.4	14.8	13.9	12.9	11.8	10.9	9.97	9.18	8.49	7.89	7.38	6.95	6.59	6.30	6.07	5.90	5.70	5.76	2.1
2.2	17.4	17.1	16.4	15.3	14.1	12.9	11.7	10.7	9.80	9.03	8.36	7.80	7.33	6.93	6.61	6.36	6.17	6.06	6.02	2.2
2.3	19.4	19.0	18.1	16.8	15.3	13.9	12.0	11.4	10.4	9.57	8.84	8.22	7.70	7.27	6.92	6.64	6.45	6.32	6.28	2.3
2.4	21.5	21.0	19.9	18.4	16.7	15.0	13.5	12.2	11.1	10.1	9.32	8.65	8.08	7.61	7.23	6.93	6.72	6.59	6.54	2.4
2.5	23.8	23.3	21.9	20.0	18.1	16.2	14.5	13.0	11.7	10.7	9.81	9.07	8.46	7.95	7.54	7.23	6.99	6.85	6.80	2.5
2.6	26.3	25.6	24.0	21.8	19.5	17.3	15.4	13.8	12.4	11.3	10.3	9.51	8.85	8.30	7.86	7.52	7.27	7.12	7.06	2.6
2.7	29.0	28.2	26.2	23.7	21.0	18.6	16.4	14.6	13.1	11.8	10.8	9.94	9.23	8.61	8.17	7.81	7.54	7.38	7.33	2.7
2.8	31.9	30.9	28.6	25.6	22.6	19.8	17.4	15.4	13.8	12.4	11.3	10.4	9.62	8.99	8.49	8.10	7.82	7.65	7.59	2.8
2.9	35.0	33.9	31.2	27.7	24.2	21.1	18.5	16.3	14.5	13.0	11.8	10.8	10.0	9.34	8.81	8.40	8.10	7.92	7.85	2.9
3.0	38.4	37.0	33.9	29.9	25.9	22.4	19.5	17.1	15.2	13.6	12.3	11.3	10.4	9.70	9.13	8.70	8.38	8.18	8.11	3.0
3.1	42.0	40.4	36.7	32.1	27.7	22.8	20.6	18.0	15.9	14.2	12.8	11.7	10.8	10.0	9.45	8.99	8.66	8.45	8.38	3.1
3.2	45.8	43.9	39.7	34.4	29.5	25.2	21.7	18.9	16.6	14.8	13.4	12.2	11.2	10.4	9.77	9.29	8.94	8.72	8.64	3.2
3.3	49.9	47.7	42.8	36.8	31.3	26.6	22.8	19.8	17.4	15.4	13.9	12.6	11.6	10.8	10.1	9.59	9.21	8.99	8.90	3.3
3.4	54.3	51.7	46.0	39.3	33.2	28.1	23.9	20.7	18.1	16.1	14.4	13.1	12.0	11.1	10.4	9.88	9.49	9.25	9.17	3.4
3.5	58.9	56.0	49.5	41.9	35.2	29.5	25.1	21.6	18.9	16.7	14.9	13.5	12.4	11.5	10.7	10.2	9.77	9.52	9.43	3.5
3.6	63.8	60.4	53.0	44.6	37.1	31.0	26.2	22.5	19.6	17.3	15.5	14.0	12.8	11.8	11.1	10.5	10.1	9.79	9.69	3.6
3.7	69.0	65.1	56.8	47.3	39.2	32.6	27.4	23.4	20.4	17.9	16.0	14.5	13.2	12.2	11.4	10.8	10.3	10.1	9.96	3.7
3.8	74.6	70.1	60.6	50.2	41.2	34.1	28.6	24.4	21.1	18.6	16.5	14.9	13.6	12.6	11.7	11.1	10.6	10.3	10.2	3.8
3.9	80.4	75.3	64.6	53.1	43.3	35.7	29.8	25.3	21.9	19.2	17.1	15.4	14.0	12.9	12.0	11.4	10.9	10.6	10.5	3.9
4.0	86.5	80.7	68.8	56.0	45.5	37.2	31.0	26.3	22.7	19.8	17.6	15.8	14.4	13.3	12.4	11.7	11.2	10.9	10.8	4.0
4.1	93.0	86.4	73.1	59.1	47.6	38.8	32.2	27.2	23.4	20.5	18.2	16.3	14.8	13.6	12.7	12.0	11.5	11.1	11.0	4.1
4.2	99.8	92.4	77.5	62.2	49.9	40.5	33.4	28.2	24.2	21.1	18.7	16.8	15.2	14.0	13.0	12.3	11.7	11.4	11.3	4.2
4.3	10.7	98.6	82.1	65.3	52.1	42.1	34.7	29.2	25.0	21.8	19.3	17.2	15.7	14.4	13.4	12.6	12.0	11.7	11.5	4.3
4.4	11.4	10.5	86.8	68.6	54.4	43.7	35.9	30.1	25.8	22.4	19.8	17.7	16.1	14.7	13.7	12.9	12.3	11.9	11.8	4.4
4.5	12.2	11.2	91.7	71.9	56.7	45.4	37.2	31.1	26.6	23.1	20.3	18.2	16.5	15.1	14.0	13.2	12.6	12.2	12.1	4.5
4.6	13.0	11.9	96.7	75.2	59.0	47.1	38.4	32.1	27.4	23.7	20.9	18.7	16.9	15.5	14.3	13.5	12.9	12.5	12.3	4.6
4.7	13.9	12.6	10.2	78.6	61.3	48.8	39.7	33.1	28.1	24.4	21.4	19.1	17.3	15.8	14.7	13.8	13.2	12.8	12.6	4.7
4.8	14.8	13.4	10.7	82.1	63.7	50.5	41.0	34.1	28.9	25.0	22.0	19.6	17.7	16.2	15.0	14.1	13.4	13.0	12.9	4.8
4.9	15.7	14.1	11.2	85.6	66.1	52.2	42.2	35.1	29.7	25.7	22.6	20.1	18.1	16.6	15.3	14.4	13.7	13.3	13.1	4.9
5.0	16.7	15.0	11.8	89.1	68.5	53.9	43.5	36.1	30.5	26.4	23.1	20.6	18.5	16.9	15.7	14.7	14.0	13.6	13.4	5.0
5.1	17.7	15.8	12.4	92.7	71.0	55.6	44.8	37.1	31.3	27.0	23.7	21.0	19.0	17.3	16.0	15.0	14.3	13.8	13.7	5.1
5.2	18.8	16.7	12.9	96.4	73.4	57.4	46.1	38.1	32.2	27.7	24.2	21.5	19.4	17.7	16.3	15.3	14.6	14.1	13.9	5.2
5.3	19.9	17.6	13.5	10.0	75.9	59.2	47.4	39.1	33.0	28.3	24.8	22.0	19.8	18.0	16.7	15.6	14.9	14.4	14.2	5.3
5.4	21.0	18.5	14.1	10.4	78.4	60.9	48.8	40.1	33.8	29.0	25.3	22.5	20.2	18.4	17.0	15.9	15.1	14.6	14.5	5.4
5.5	22.2	19.4	14.7	10.8	80.9	62.7	50.1	41.1	34.6	29.7	25.9	22.9	20.6	18.8	17.3	16.2	15.4	14.9	14.7	5.5
5.6	23.4	20.4	15.3	11.1	83.5	64.5	51.4	42.2	35.4	30.3	26.4	23.4	21.0	19.1	17.7	16.5	15.7	15.2	15.0	5.6
5.7	24.7	21.4	16.0	11.5	86.0	66.3	52.7	43.2	36.2	31.0	27.0	23.9	21.5	19.5	18.0	16.8	16.0	15.5	15.3	5.7
5.8	26.0	22.4	16.6	11.9	88.6	68.1	54.1	44.2	37.0	31.7	27.6	24.4	21.9	19.9	18.3	17.2	16.3	15.7	15.5	5.8
5.9	27.4	23.5	17.3	12.3	91.2	69.9	55.4	45.2	37.9	32.3	28.1	24.9	22.3	20.3	18.7	17.5	16.6	16.0	15.8	5.9
6.0	28.8	24.6	17.9	12.7	93.8	71.7	56.8	46.3	38.7	33.0	28.7	25.3	22.7	20.6	19.0	17.8	16.8	16.3	16.1	6.0
6.1	30.3	25.7	18.6	13.1	96.4	73.6	58.1	47.3	39.5	33.7	29.3	25.8	23.1	21.0	19.3	18.1	17.1	16.5	16.3	6.1
6.2	31.8	26.9	19.3	13.5	99.1	75.4	59.5	48.4	40.3	34.4	29.8	26.3	23.6	21.4	19.7	18.4	17.4	16.8	16.6	6.2
6.3	33.4	28.0	20.0	13.9	10.2	77.2	60.8	49.4	41.2	35.0	30.4	26.8	24.0	21.7	20.0	18.7	17.7	17.1	16.9	6.3
6.4	35.0	29.3	20.7	14.3	10.4	79.1	62.2	50.4	42.0	35.7	31.0	27.3	24.4	22.1	20.3	19.0	18.0	17.4	17.1	6.4
6.5	36.6	30.5	21.4	14.8	10.7	81.0	63.5	51.5	42.8	36.4	31.5	27.8	24.8	22.5	20.7	19.3	18.3	17.6	17.4	6.5
6.6	38.3	31.8	22.1	15.2	11.0	82.8	64.9	52.5	43.6	37.1	32.1	28.2	25.2	22.9	21.0	19.6	18.5	17.9	17.7	6.6
6.7	40.1	33.1	22.9	15.6	11.2	84.7	66.3	53.6	44.5	37.8	32.7	28.7	25.7	23.2	21.3	19.9	18.8	18.2	17.9	6.7
6.8	41.9	34.4	23.6	16.0	11.5	86.6	67.7	54.6	45.3	38.4	33.2	29.2	26.1	23.6	21.7	20.2	19.1	18.4	18.2	6.8
6.9	43.8	35.7	24.4	16.5	11.8	88.5	69.0	55.7	46.1	39.1	33.8	29.7	26.5	24.0	22.0	20.5	19.4	18.7	18.4	6.9
7.0	45.7	37.1	25.1	16.9	12.1	90.4	70.4	56.7	47.0	39.8	34.4	30.2	26.9	24.3	22.3	20.8	19.7	19.0	18.7	7.0
7.1	47.7	38.5	25.9	17.3	12.3	92.2	71.8	57.8	47.8	40.5	34.9	30.7	27.3	24.7	22.7	21.1	20.0	19.3	19.0	7.1
7.2	49.8	40.0	26.7	17.7	12.6	94.1	73.2	58.9	48.7	41.2	35.5	31.1	27.8	25.1	23.0	21.4	20.3	19.5	19.2	7.2
7.3	51.9	41.5	27.5	18.2	12.9	96.0	74.6	59.9	49.5	41.8	36.1	31.6	28.2	25.5	23.4	21.7	20.5	19.8	19.5	7.3
7.4	54.0	43.0	28.3	18.6	13.2	98.0	76.0	61.0	50.3	42.5	36.6	32.1	28.6	25.8	23.7	22.1	20.8	20.1	19.8	7.4
7.5	56.3	44.5	29.1	19.1	13.5	99.9	77.4	62.0	51.2	43.2	37.2	32.6	29.0	26.0	24.0	22.4	21.1	20.3	20.0	7.5
7.6	58.5	46.0	29.9	19.5	13.7	10.2	78.8	63.1	52.0	43.9	37.8	33.1	29.4	26.6	24.4	22.7	21.4	20.6	20.3	7.6
7.7	60.9	47.6	30.7	20.0	14.0	10.4	80.2	64.2	52.8	44.6	38.4	33.6	29.9	27.0	24.7	23.0	21.7	20.9	20.6	7.7
7.8	63.3	49.3	31.5	20.4	14.3	10.6	81.6	65.2	53.7	45.3	38.9	34.1	30.3	27.3	25.0	23.3	22.0	21.2	20.8	7.8
7.9	65.7	50.9	32.4	20.9	14.6	10.8	83.0	66.3	54.5	45.9	39.5	34.6	30.7	27.7	25.4	23.6	22.3	21.4	21.1	7.9
8.0	68.3	52.6	33.4	21.3	14.9	11.0	84.4	67.4	55.4	46.6	40.1	35.0	31.1	28.1	25.7	23.9	22.5	21.7	21.4	8.0
8.1	70.9	54.3	34.1	21.8	15.2	11.1	85.8	68.4	56.2	47.3	40.6	35.5	31.6	28.5	26.0	24.2	22.8	22.0	21.6	8.1
8.2	73.5	56.0	34.9	22.2	15.4	11.3	87.2	69.5	57.1	48.0	41.2	36.0	32.0	28.8	26.4	24.5	23.1	22.2	21.9	8.2
8.3	76.2	57.8	35.8	22.7	15.7	11.5	88.6	70.6	57.9	48.7	41.8	36.5	32.4	29.2	26.7	24.8	23.4	22.5	22.2	8.3
8.4	79.0	59.6	36.7	23.2	16.0	11.7	90.1	71.7	58.8	49.4	42.4	37.0	32.9	29.6	27.1	25.1	23.7	22.8	22.4	8.4
8.5	81.9	61.4	37.5	23.6	16.3	11.9	91.5	72.7	59.6	50.1	42.9	37.5	33.3	30.0	27.4	25.4	24.0	23.1	22.7	8.5
8.6	84.8	63.2	38.4	24.1	16.6	12.1	92.9	73.8	60.4	50.7	43.5	38.0	33.7	30.3	27.8	25.7	24.3	23.3	23.0	8.6
8.7	87.8	65.1	39.3	24.6	16.9	12.3	94.3	74.9	61.3	51.4	44.1	38.5	34.1	30.7	28.1	26.1	24.5	23.6	23.2	8.7
8.8	90.9	67.0	40.2	25.0	17.2	12.5	95.8	76.0	62.1	52.1	44.7	39.0	34.6	31.1	28.4	26.4	24.8	23.9	23.5	8.8
8.9	94.0	69.0	41.1	25.5	17.5	12.7	97.2	77.1	63.0	52.8	45.2	39.5	35.0	31.5	28.7	26.7	25.1	24.2	23.8	8.9
9.0	97.2	70.9	42.0	26.0	17.8	12.9	98.6	78.2	63.9	53.5	45.8	39.9	35.4	31.8	29.1	27.0	25.4	24.4	24.0	9.0
9.1	1005	72.9	42.9	26.4	18.0	13.1	10.0	79.3	64.7	54.2	46.4	40.4	35.8	32.2	29.4	27.3	25.7	24.7	24.3	9.1
9.2	1038	74.9	43.9	26.9	18.3	13.3	10.1	80.4	65.6	54.9	46.9	40.9	36.2	32.6	29.7	27.6	26.0	25.0	24.6	9.2
9.3	1072	77.0	44.8	27.4	18.6	13.5	10.3	81.5	66.4	55.6	47.5	41.3	36.6	32.9	30.1	27.9	26.3	25.2	24.9	9.3
9.4	1107	79.1	45.7	27.9	18.9	13.7	10.4	82.6	67.3	56.3	48.0	41.8	37.0	33.3	30.4	28.2	26.5	25.5	25.1	9.4
9.5	1143	81.2	46.7	28.4	19.2	13.9	10.6	83.7	68.2	57.0	48.6	42.2	37.4	33.6	30.7	28.5	26.8	25.8	25.4	9.5
9.6	1180	83.3	47.6	28.8	19.5	14.1	10.7	84.8	69.1	57.7	49.1	42.7	37.7	33.9	31.0	28.8	27.1	26.1	25.6	9.6
9.7	1217	85.5	48.5	29.3	19.8	14.3	10.9	85.0	69.9	58.4	49.6	43.1	38.1	34.3	31.4	29.1	27.4	26.3	25.9	9.7
9.8	1255	87.7	49.5	29.8	20.1	14.5	11.0	87.0	70.8	59.1	50.2	43.5	38.4	34.6	31.7	29.5	27.7	26.6	26.2	9.8
9.9	1294	89.9	50.5	30.3	20.4	14.7	11.2	88.2	71.7	59.8	50.7	43.9	38.7	34.9	32.0	29.8	28.0	26.9	26.4	9.9
10.0	1333	92.1	51.4	30.8	20.7	14.9	11.3	89.3	72.6	60.5	51.2	44.2	39.0	35.2	32.3	30.1	38.3	27.2	26.7	10.0

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